Chapter 3

Perpendicular and Parallel Lines

Section 3
Parallel Lines and Transversals

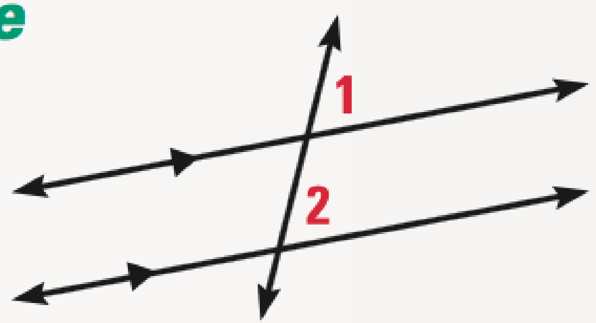
GOAL 1: Properties of Parallel Lines

In the activity of page 142, you may have discovered the following results.

POSTULATE

POSTULATE 15 ***Corresponding Angles Postulate***

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



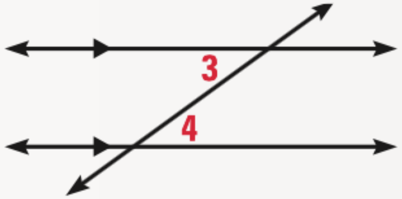
$\angle 1 \cong \angle 2$

We will prove Theorems 3.5, 3.6, and 3.7 in Exercises 27-29.

THEOREMS ABOUT PARALLEL LINES

THEOREM 3.4 ***Alternate Interior Angles***

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

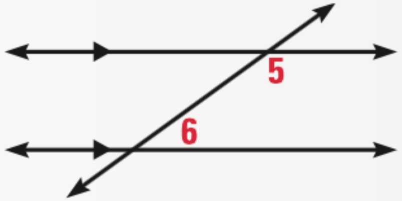


$\angle 3 \cong \angle 4$

(same-side)

THEOREM 3.5 ***Consecutive Interior Angles***

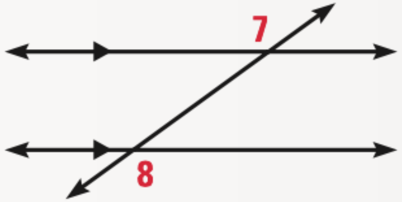
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.



$m\angle 5 + m\angle 6 = 180°$

THEOREM 3.6 ***Alternate Exterior Angles***

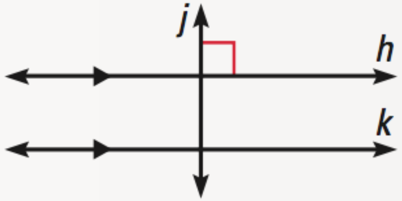
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.



$\angle 7 \cong \angle 8$

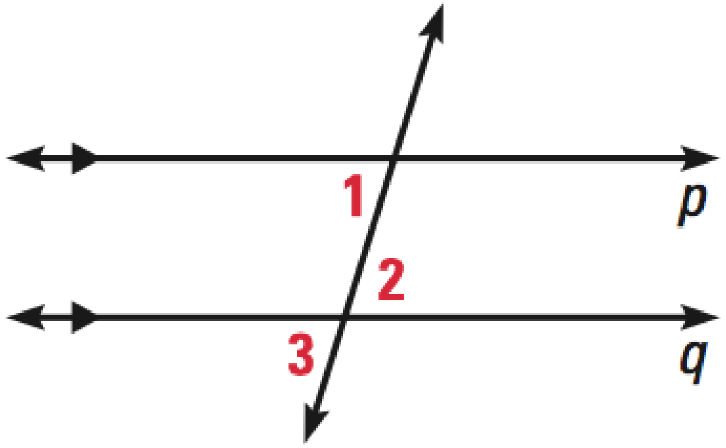
THEOREM 3.7 ***Perpendicular Transversal***

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.



$j \perp k$

Example 1: Proving the Alternate Interior Angles Theorem



Prove the Alternate Interior Angles Theorem.

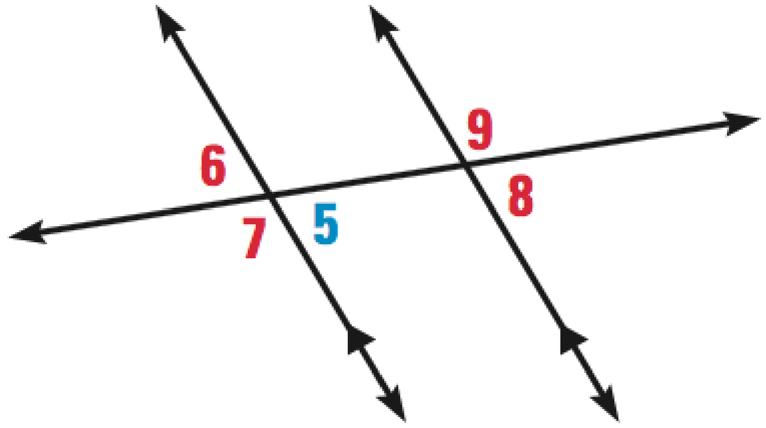
Given: p || q

Prove: <1 cong. <2

Statements	Reasons
1 – p || q	Given
2 - <1 cong. ~~<3~~	Corr. <'s Postulate (Postulate 15)
3 – ~~<3~~ cong. <2	Vertical Angles Theorem
4 - <1 cong. <2	Transitive

Example 2: Using Properties of Parallel Lines

Given that m<5 = 65°, find each measure. Tell which postulate or theorem you use.



a. m<6 = 65* (Vertical Angles Theorem)

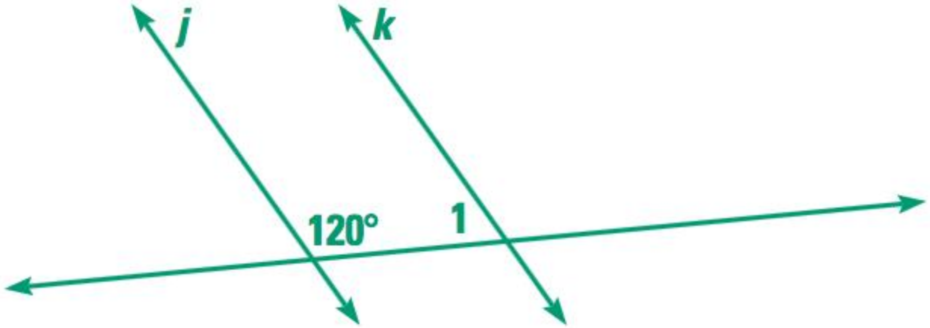
b. m<7 = 180 – 65 = 115* (Linear Pair Postulate)

c. m<8 = 65* (Alt. Ext. w/ <6 OR Corresponding w/ <5)

d. m<9 = 115* (Alt. Ext. w/ <7 OR Linear Pair w/ <8)

Example 3: Classifying Leaves

Botany: Some plants are classified by the arrangement of the veins in their leaves. In the diagram of the leaf, *j* || *k*. What is m<1?

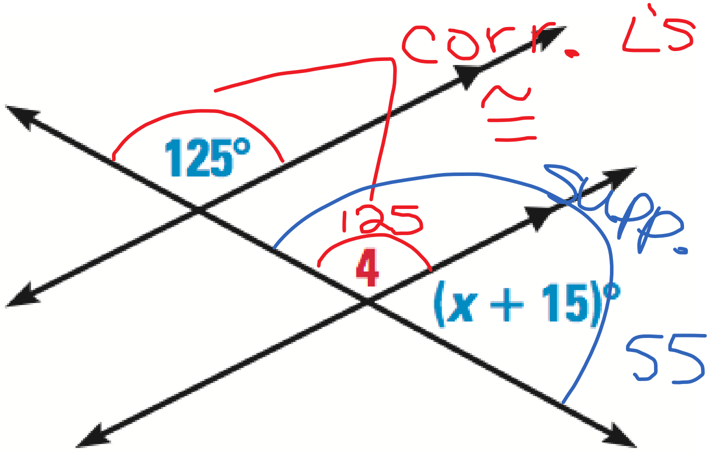


SSI → supplementary → 180 – 120 = 60

m<1 = 60*

GOAL 2: Properties of Special Pairs of Angles

Example 4: Using Properties of Parallel Lines



Use properties of parallel lines to find the value of x.

1

$x + 15 = 55$

$x = 40$

2 $125 + x + 15 = 180$

$x + 140 = 180$

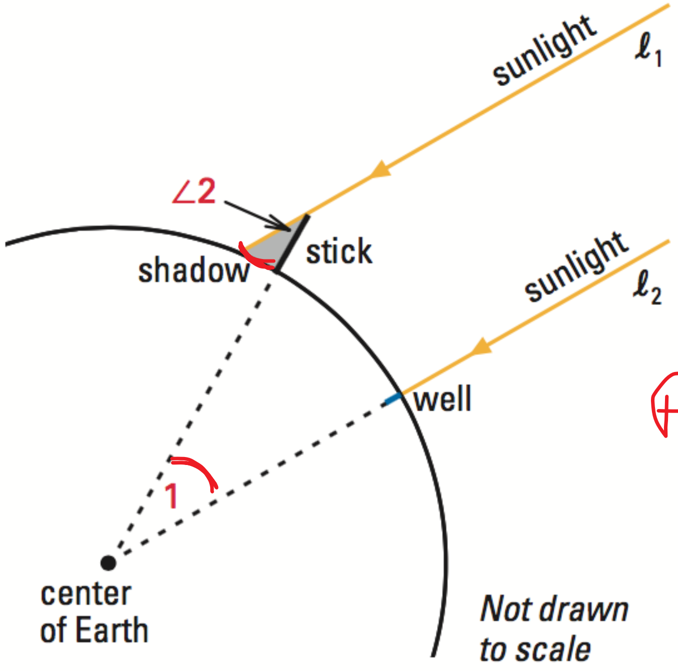
$x = 40$

Example 5: Estimating Earth's Circumference

HISTORY CONNECTION Eratosthenes was a Greek scholar. Over 2000 years ago, he estimated Earth's circumference by using the fact that the Sun's rays are parallel.

Eratosthenes chose a day when the Sun shone exactly down a vertical well in Syene at noon. On that day, he measured the angle the Sun's rays made with a vertical stick in Alexandria at noon. He discovered that

$$m\angle 2 \approx \frac{1}{50} \text{ of a circle.}$$



A.I. ∠'s
↳always ≅

By using properties of parallel lines, he knew that $m\angle 1 = m\angle 2$. So he reasoned that

$$m\angle 1 \approx \frac{1}{50} \text{ of a circle.}$$

At the time, the distance from Syene to Alexandria was believed to be 575 miles.

$$\frac{1}{50} \text{ of a circle} \approx \frac{575 \text{ miles}}{\text{Earth's circumference}}$$

Earth's circumference ≈ 50(575 miles) ← **Use cross product property.**

≈ 29,000 miles

How did Eratosthenes know that $m\angle 1 = m\angle 2$?